

9ª LISTA DE EXERCÍCIOS

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1. Construir o gráfico cartesiano das funções:

(a) $y = -\frac{1}{x}$

(e) $y = \frac{1}{x+1}$

(i) $f(x) = \frac{x+3}{x+2}$

(b) $y = \frac{1}{2x}$

(f) $f(x) = \frac{1}{x-1}$

(j) $f(x) = \frac{x+1}{x-1}$

(c) $y = -\frac{1}{2x}$

(g) $f(x) = \frac{1}{2-x}$

(k) $f(x) = \frac{x-1}{2-x}$

(d) $y = \frac{1}{|x|}$

(h) $f(x) = \frac{1}{|x+2|}$

(l) $f(x) = \left| \frac{x-1}{x} \right|$

2. Sejam as funções reais f e g , definidas por $f(x) = x^2 + 4x - 5$ e $g(x) = 2x - 3$.

(a) Obtenha as leis que definem $f \circ g$ e $g \circ f$.

(b) Calcule $(f \circ g)(2)$ e $(g \circ f)(2)$.

(c) Determine os valores do domínio da função $f \circ g$ que produzem imagem 16.

3. Sejam as funções reais f e g , definidas por $f(x) = x^2 - x - 2$ e $g(x) = 1 - 2x$.

(a) Obtenha as leis que definem $f \circ g$ e $g \circ f$.

(b) Calcule $(f \circ g)(-2)$ e $(g \circ f)(-2)$.

(c) Determine os valores do domínio da função $f \circ g$ que produzem imagem 10.

4. Sejam as funções reais f e g definidas por $f(x) = x^2 - 4x + 1$ e $g(x) = x^2 - 1$. Obtenha as leis que definem $f \circ g$ e $g \circ f$.

5. Sejam as funções reais f e g definidas por $f(x) = 2$ e $g(x) = 3x - 1$. Obtenha as leis que definem $f \circ g$ e $g \circ f$.

6. Nas funções reais f e g , definidas por $f(x) = x^2 + 2$ e $g(x) = x - 3$, obtenha as leis que definem:

(a) $f \circ g$

(c) $f \circ f$

(b) $g \circ f$

(d) $g \circ g$.

7. Considere a função em \mathbb{R} definida por $f(x) = x^3 - 3x^2 + 2x - 1$. Qual é a lei que define $f(-x)$?
E $f\left(\frac{1}{x}\right)$? E $f(x-1)$?

8. Dadas as funções reais definidas por $f(x) = 3x + 2$ e $g(x) = 2x + a$, determine o valor de a de modo que se tenha $f \circ g = g \circ f$.

9. Se $f(x) = x^3$ e $g(x) = x^4$, mostre que $f \circ g = g \circ f$.

10. Sejam as funções $f(x) = x^2 + 2x + 3$ e $g(x) = x^2 + ax + b$. Mostre que, se $f \circ g = g \circ f$, então $f = g$.

11. Sejam as funções definidas por $f(x) = \sqrt{x}$ e $g(x) = x^2 - 3x - 4$. Determine os domínios das funções $f \circ g$ e $g \circ f$.
12. Sejam as funções definidas por $f(x) = \sqrt{x-1}$ e $g(x) = 2x^2 - 5x + 3$. Determine os domínios das funções $f \circ g$ e $g \circ f$.
13. Sejam as funções definidas por $f(x) = \frac{x+1}{x-2}$, definida para todo x real e $x \neq 2$, e $g(x) = 2x+3$, definida para todos x real. Forneça:
- o domínio da função $f \circ g$;
 - o domínio da função $g \circ f$.
14. Sejam as funções reais $f(x) = 2x + 1$, $g(x) = x^2 - 1$ e $h(x) = 3x + 2$. Obtenha a lei que define $(h \circ g) \circ f$.
15. Sejam as funções reais $f(x) = 1 - x$, $g(x) = x^2 - x + 2$ e $h(x) = 2x + 3$. Obtenha a lei que define $h \circ (g \circ f)$.
16. Dadas $f(x) = 3$ e $g(x) = x^2$, determine $f(g(x))$.
17. Se $f(x) = \frac{1}{1-x}$, determine $(f \circ [f \circ f])(x)$.
18. Dadas as funções f , g e h , de \mathbb{R} em \mathbb{R} , definidas por $f(x) = 3x$, $g(x) = x^2 - 2x + 1$ e $h(x) = x + 2$, obtenha $((hof)og)(2)$.
19. Nas funções bijetoras abaixo, de \mathbb{R} em \mathbb{R} , obtenha a lei de correspondência que define a função inversa.
- | | | |
|-----------------------------|--------------------------|------------------------------|
| (a) $f(x) = 2x + 3$ | (c) $h(x) = x^3 + 2$ | (f) $r(x) = \sqrt[3]{x-1}$ |
| (b) $g(x) = \frac{4x-1}{3}$ | (d) $p(x) = (x-1)^3 + 2$ | (g) $s(x) = \sqrt[3]{1-x^3}$ |
| (e) $q(x) = \sqrt[3]{x+2}$ | | |
20. O gráfico de uma função f é o segmento de reta que une os pontos $(-3, 4)$ e $(3, 0)$. Se f^{-1} é a função inversa de f , determine $f^{-1}(2) = 0$.
21. Dada a função $f : \mathbb{R} \rightarrow \mathbb{R}$, bijetora, definida por $f(x) = x^3 + 1$, determine sua inversa
22. Obtenha a função inversa das seguintes funções:
- $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{1\}$, $f(x) = \frac{x+3}{x-3}$
 - $f : \mathbb{R} - \{-1\} \rightarrow \mathbb{R} - \{2\}$, $f(x) = \frac{2x+3}{x+1}$
 - $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{-1\}$, $f(x) = \frac{4-x}{x-3}$
 - $f : \mathbb{R} - \{1/3\} \rightarrow \mathbb{R} - \{5/3\}$, $f(x) = \frac{5x+2}{3x-1}$
 - $f : \mathbb{R}^* \rightarrow \mathbb{R} - \{4\}$, $f(x) = \frac{4x+2}{x}$
 - $f : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{3\}$, $f(x) = \frac{3x+2}{x-3}$

23. Nas funções que seguem, construa num mesmo plano cartesiano os gráficos de f e f^{-1} .

(a) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + 1$

(b) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = \frac{2x + 2}{3}$

(c) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 1 - x^3$

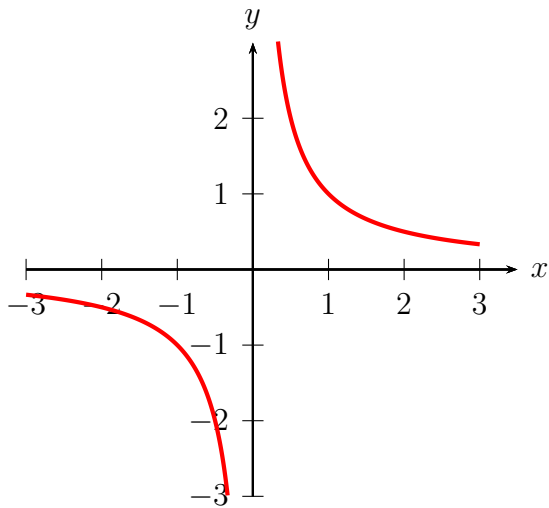
(d) $f : \mathbb{R}_- \rightarrow B, B = \{y \in \mathbb{R} | y \leq 1\}, f(x) = 1 - x^2$

(e) $f : A \rightarrow A, A = \{y \in \mathbb{R} | x \geq -1\}, f(x) = x^2 + 2x,$

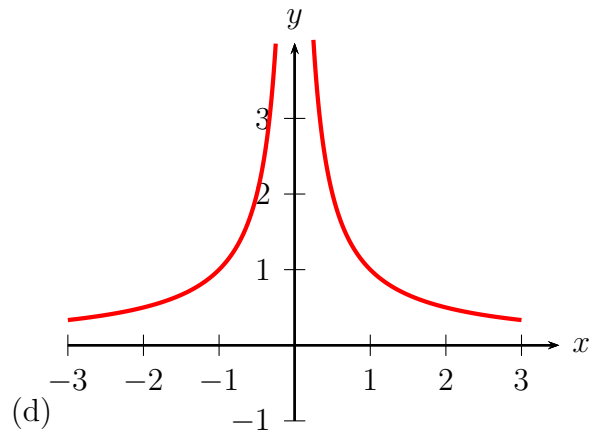
(f) $f : \mathbb{R}^* \rightarrow \mathbb{R}^*, f(x) = \frac{1}{x}$

(g) $f : \mathbb{R}^* \rightarrow \mathbb{R} - \{1\}, f(x) = \frac{x - 1}{x}$

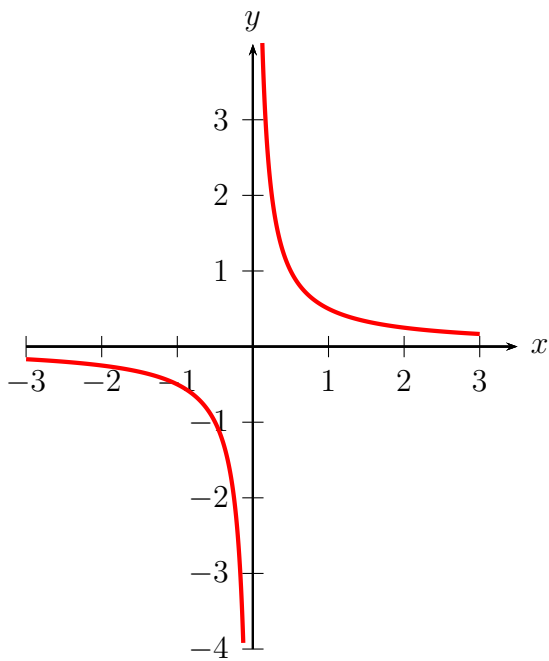
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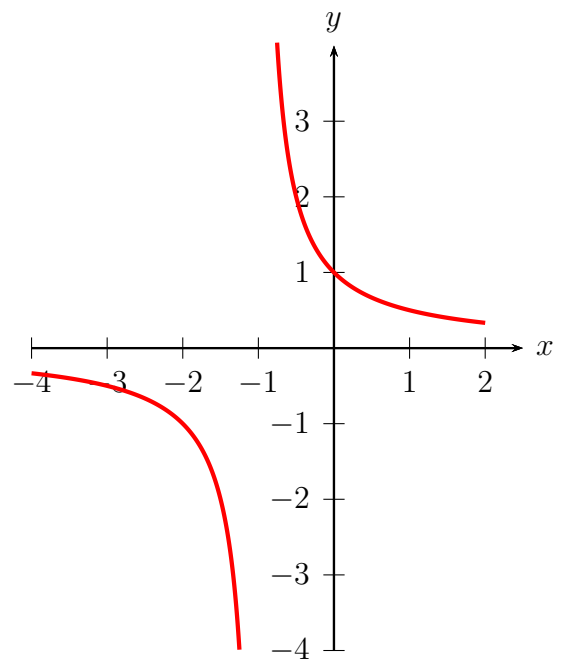
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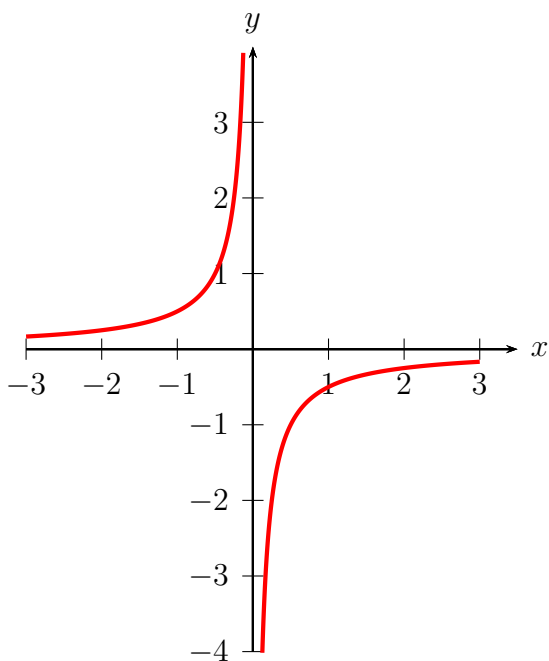
(d)



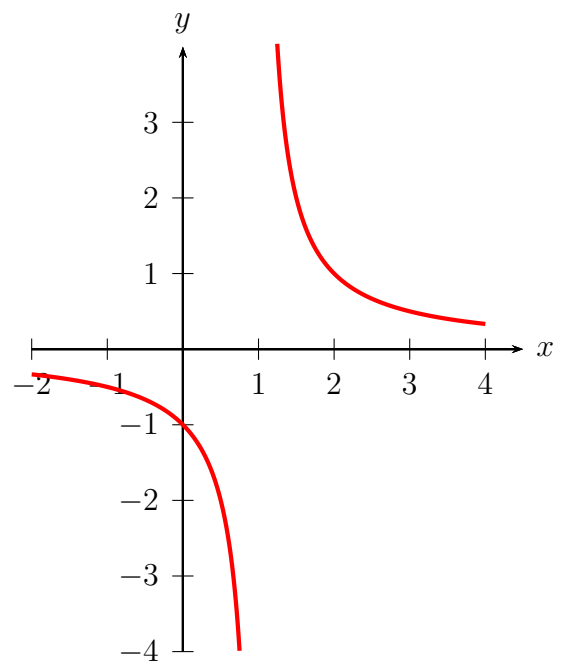
(b)



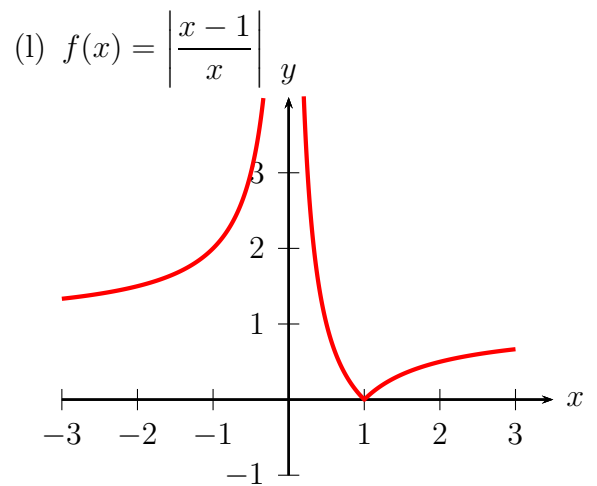
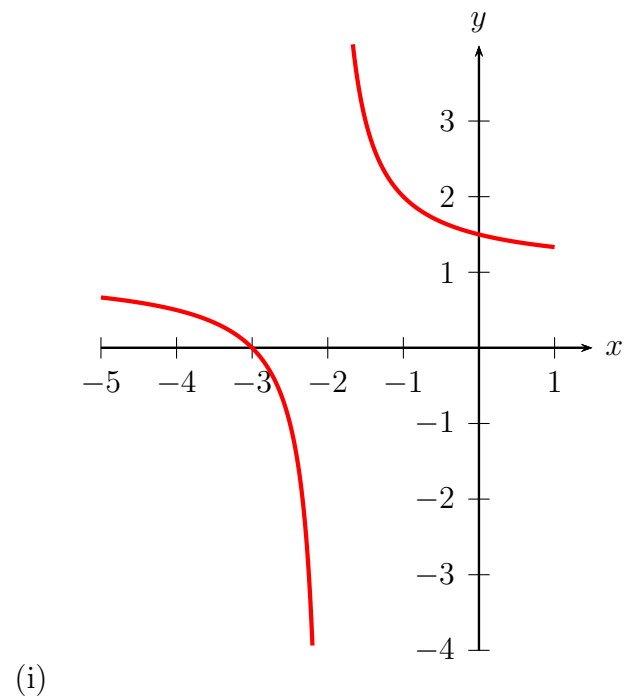
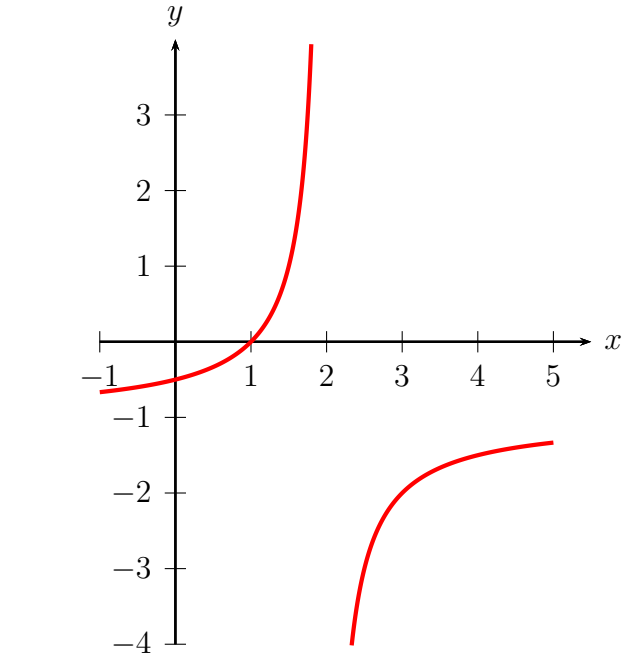
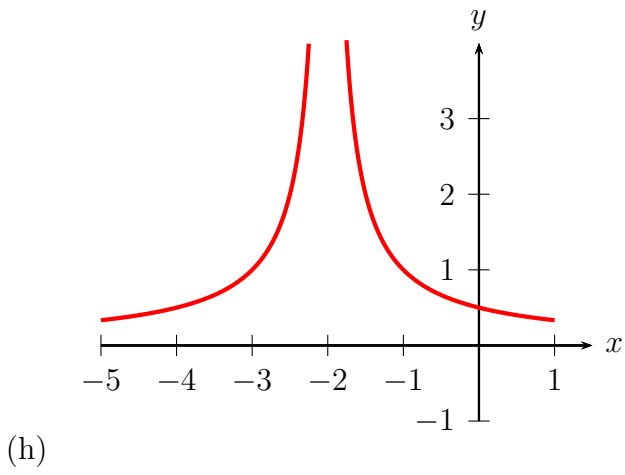
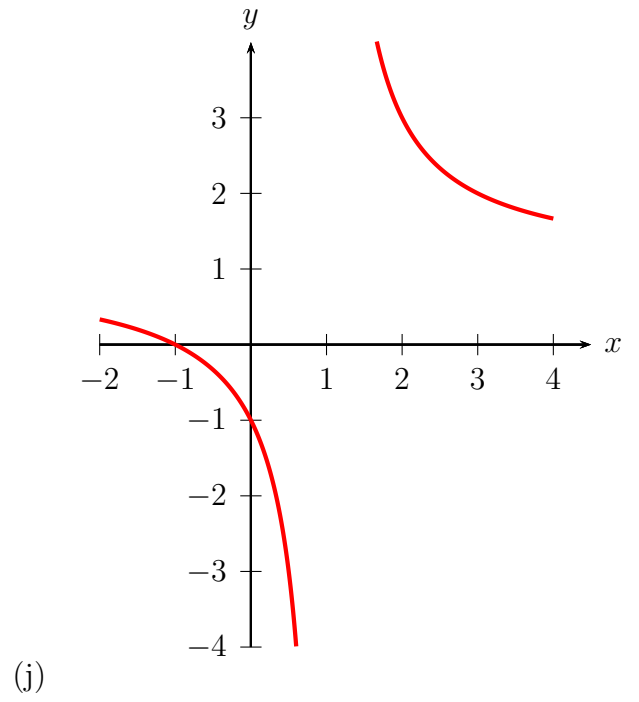
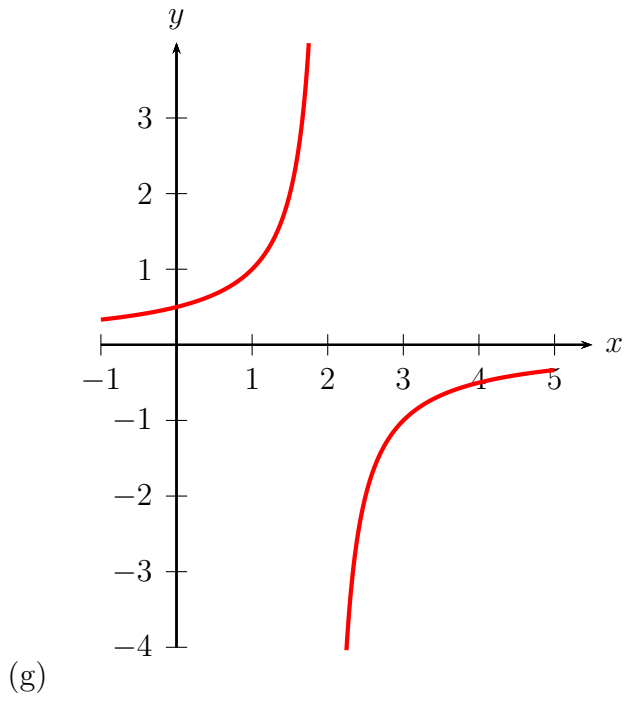
(e)



(c)



(f)



2. (a) $(f \circ g)(x) = 4x^2 - 4x - 8$; $(g \circ f)(x) = 2x^2 + 8x - 13$
 (b) $(f \circ g)(2) = 0$; $(g \circ f)(2) = 11$.
 (c) $S = \{-2, 3\}$
3. (a) $(f \circ g)(x) = 4x^2 - 2x - 2$; $(g \circ f)(x) = -2x^2 + 2x + 5$.
 (b) $(f \circ g)(-2) = 18$; $(g \circ f)(-2) = -7$.
 (c) $S = \left\{-\frac{3}{2}, 2\right\}$.
4. $(f \circ g)(x) = x^4 - 6x^2 + 6$; $(g \circ f)(x) = x^4 - 8x^3 + 18x^2 - 8x$.
5. $(f \circ g)(x) = 2$; $(g \circ f)(x) = 5$.
6. (a) $(f \circ g)(x) = x^2 - 6x + 11$ (c) $(f \circ f)(x) = x^4 + 4x^2 + 6$
 (b) $(g \circ f)(x) = x^2 - 1$ (d) $(g \circ g)(x) = x - 6$
7. $f(-x) = -x^3 - 3x^2 - 2x - 1$; $f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{3}{x^2} + \frac{2}{x} - 1$; $f(x-1) = x^3 - 6x^2 + 11x - 7$.
8. $a = 1$
9. Demonstração.
10. Demonstração.
11. $Dom(f \circ g) = \{x \in \mathbb{R} | x \leq -1 \text{ ou } x \geq 3\}$; $(g \circ f)(x) = \{x \in \mathbb{R} | x \geq 0\}$.
12. $(f \circ g)(x) = \{x \in \mathbb{R} | x \leq \frac{1}{2} \text{ ou } x \geq 2\}$; $Dom(g \circ f) = \{x \in \mathbb{R} | x \geq 1\}$.
13. (a) $Dom(f \circ g) = \{x \in \mathbb{R} | x \neq -\frac{1}{2}\}$
 (b) $Dom(g \circ f) = \{x \in \mathbb{R} | x \neq 2\}$
14. $((h \circ g) \circ f)(x) = 12x^2 + 12x + 2$
15. $(h \circ (g \circ f))(x) = 2x^2 - 2x + 7$.
16. $f(g(x)) = 3$
17. $(f \circ [f \circ f])(x) = x$.
18. $((hof)og)(2) = 5$.
19. Nas funções bijetoras abaixo, de \mathbb{R} em \mathbb{R} , obtenha a lei de correspondência que define a função inversa.
- (a) $f^{-1}(x) = \frac{x-3}{2}$ (c) $h^{-1}(x) = \sqrt[3]{x-2}$ (f) $r^{-1}(x) = x^3 + 1$
 (b) $g^{-1}(x) = \frac{3x+1}{4}$ (d) $p^{-1}(x) = \sqrt[3]{x-2} + 1$ (g) $s^{-1}(x) = \sqrt[3]{1-x^3}$
 (e) $q^{-1}(x) = x^3 - 2$
20. $f^{-1}(2) = 0$.
21. $f^{-1}(x) = \sqrt[3]{x-1}$
22. (a) $f^{-1} : \mathbb{R} - \{1\} \rightarrow \mathbb{R} - \{3\}$, $f(x) = \frac{3x+3}{x-1}$

(b) $f^{-1} : \mathbb{R} - \{2\} \rightarrow \mathbb{R} - \{-1\}$, $f(x) = \frac{-x + 3}{x - 1}$

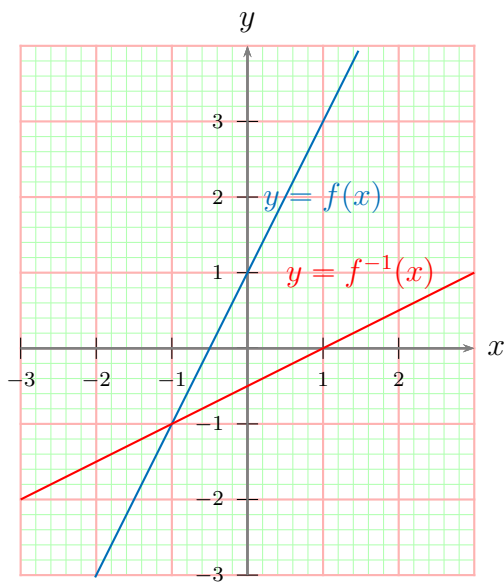
(c) $f^{-1} : \mathbb{R} - \{-1\} \rightarrow \mathbb{R} - \{3\}$, $f(x) = \frac{3x + 4}{x + 1}$

(d) $f^{-1} : \mathbb{R} - \{5/3\} \rightarrow \mathbb{R} - \{1/3\}$, $f(x) = \frac{x + 2}{3x - 5}$

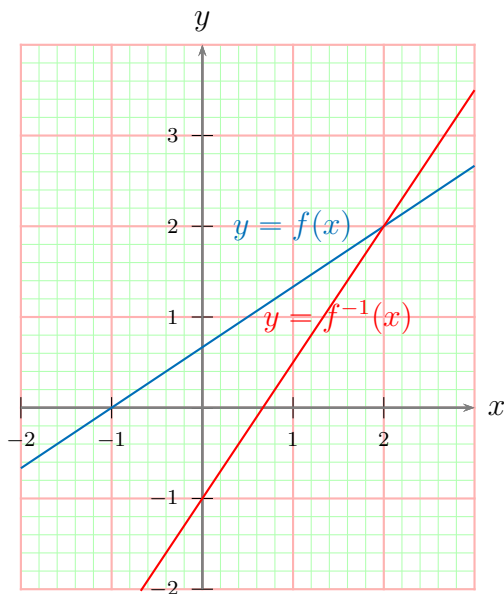
(e) $f^{-1} : \mathbb{R} - \{4\} \rightarrow \mathbb{R}^*$, $f(x) = \frac{2}{x - 4}$

(f) $f^{-1} : \mathbb{R} - \{3\} \rightarrow \mathbb{R} - \{3\}$, $f(x) = \frac{3x + 2}{x - 3}$

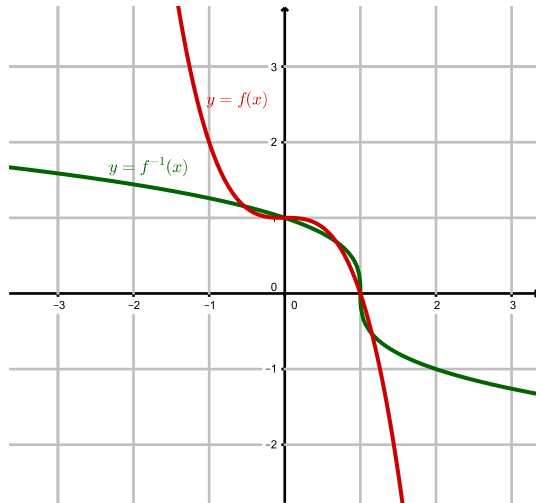
23. (a) $f^{-1} : \mathbb{R} \rightarrow \mathbb{R}$, $f^{-1}(x) = \frac{x - 1}{2}$



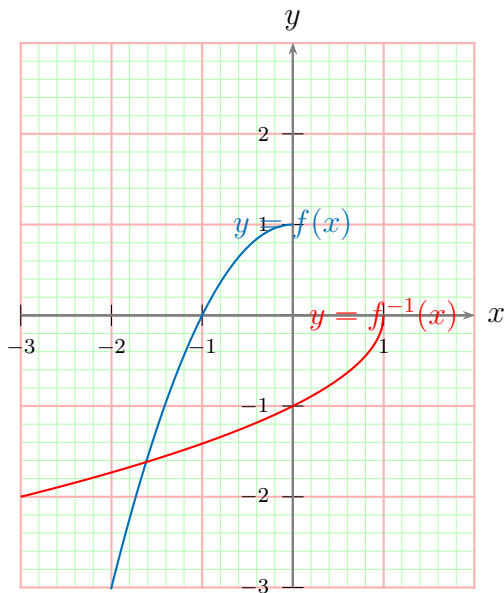
(b) $f^{-1} : \mathbb{R} \rightarrow \mathbb{R}$, $f^{-1}(x) = \frac{3x - 2}{2}$



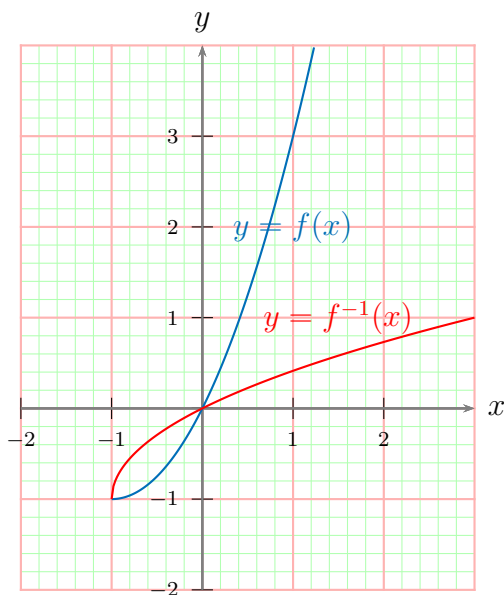
(c) $f^{-1} : \mathbb{R} \rightarrow \mathbb{R}, f^{-1}(x) = \sqrt[3]{1-x}$



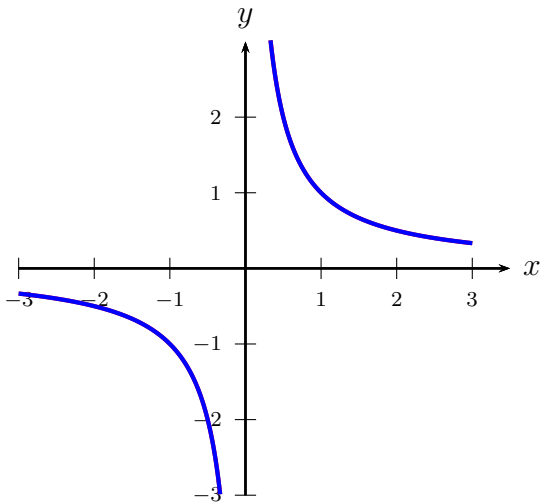
(d) $f^{-1} : B \rightarrow \mathbb{R}_-, B = \{y \in \mathbb{R} | y \leq 1\}, f^{-1}(x) = -\sqrt{1-x}$



(e) $f : A \rightarrow A, A = \{y \in \mathbb{R} | x \geq -1\} f^{-1}(x) = \sqrt{x+1} - 1,$



$$(f) f^{-1} : \mathbb{R}^* \rightarrow \mathbb{R}^*, f^{-1}(x) = \frac{1}{x}$$



$$(g) f : \mathbb{R} - \{1\} \rightarrow \mathbb{R}^*, f(x) = -\frac{1}{x-1}$$

