

7ª LISTA DE EXERCÍCIOS

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1. Estude os sinais das funções definidas em \mathbb{R} :

(a) $y = 2x + 3$

(f) $y = \frac{x}{3} + \frac{3}{2}$

(b) $y = -3x + 2$

(c) $y = 4 - x$

(g) $y = 2x - \frac{4}{3}$

(d) $y = 5 + x$

(e) $y = 3 - \frac{x}{2}$

(h) $y = -x$

2. Seja a função de \mathbb{R} em \mathbb{R} definida por $f(x) = 4x - 5$. Determine os valores do domínio da função que produzem imagens maiores que 2.

Solução: Queremos determinar os valores de x tais que

$$4x - 5 > 2.$$

Então,

$$4x > 2 + 5 \implies 4x > 7 \implies x > \frac{7}{4}.$$

3. Para que valores do domínio da função de \mathbb{R} em \mathbb{R} definida por $f(x) = \frac{3x - 1}{2}$ a imagem é menor que 4?

4. Para que valores de $x \in \mathbb{R}$ a função $f(x) = \frac{2}{3} - \frac{x}{2}$ é negativa?

5. Sejam as funções $f(x) = 2x + 3$, $g(x) = 2 - 3x$ e $h(x) = \frac{4x - 1}{2}$ definidas em \mathbb{R} . Para que valores de $x \in \mathbb{R}$, tem-se:

(a) $f(x) \geq g(x)$?

(b) $g(x) < h(x)$?

(c) $f(x) \geq h(x)$?

6. Resolva as inequações, em \mathbb{R} :

(a) $4x + 5 > 2x - 3$

(b) $5(x + 3) - 2(x + 1) \leq 2x + 3$

(c) $3(x + 1) - 2 \geq 5(x - 1) - 3(2x - 1)$

7. Resolva, em \mathbb{R} , a inequação:

$$\frac{x + 2}{3} - \frac{x - 1}{2} \geq x.$$

Solução: Veja que o $mmc(3, 2) = 6$, então multiplicando ambos os lados da desigualdade teremos

$$2(x+2) - 3(x-1) \geq 6x \implies 4x+4 - 3x+3 \geq 6x \implies -x+7 \geq 6x \implies -7x \geq -7 \implies x \leq 1.$$

Portanto, $S = \{x \in \mathbb{R} | x \leq 1\}$

8. Resolva as inequações, em \mathbb{R} :

- (a) $\frac{x-1}{2} - \frac{x-3}{4} \geq 1$
 (b) $\frac{2x-3}{2} - \frac{5-3x}{3} < 3x - \frac{1}{6}$
 (c) $(3x+1)(2x+1) \leq (2x-1)(3x+2) - (4-5x)$
 (d) $(3x-2)^2 - (3x-1)^2 > (x_2)^2 - (x-1)^2$
 (e) $4(x-2) - (3x+2) > 5x-6 - 4(x-1)$
 (f) $6(x+2) - 2(3x+2) > 2(3x-1) - 3(2x+1)$

9. Resolva, em \mathbb{R} , a inequação:

$$\frac{2x-3}{x-1} \leq 2.$$

Solução: A inequação proposta é equivalente a $\frac{2x-3}{x-1} - 2 \leq 0$. Assim para obter um mesmo denominador,

$$\frac{2x-3}{x-1} - 2\frac{x-1}{x-1} \leq 0 \iff \frac{2x-3-(2(x-1))}{x-1} \leq 0 \iff \frac{-1}{x-1} \leq 0.$$

Observe que o numerador é sempre negativo. Logo, para o desejado, precisamos que

$$x-1 > 0 \iff x > 1.$$

Portanto, $= \{x \in \mathbb{R} | x > 1\}$.

10. Resolva, em \mathbb{R} , as inequações:

- (a) $\frac{3x-2}{1-x} \leq -3$
 (b) $\frac{4x-5}{2x-1} \geq 2$
 (c) $\frac{-4-3x}{3x+2} < -1$

11. Resolva, em \mathbb{R} , as inequações:

- (a) $-2 < 3x-1 < 4$
 (b) $-4 < 4-2x \leq 3$
 (c) $-3 < 3x-2 < x$
 (d) $x+1 \leq 7-3x < \frac{x}{2} - 1$
 (e) $3x+4 < 5 < 6-2x$
 (f) $2-x < 3x+2 < 4x+1$

12. Resolva, em \mathbb{R} , os sistemas de inequações:

- (a) $\begin{cases} 3-2x \leq 1 \\ 3x-1 \leq 5 \end{cases}$
 (b) $\begin{cases} 3x-2 > 4x+1 \\ 5x+1 \leq 2x-5 \end{cases}$
 (c) $\begin{cases} 5-2x < 0 \\ 3x+1 \geq 4x-5 \\ x-3 \geq 0 \end{cases}$
 (d) $\begin{cases} 3x+2 \geq 5x-2 \\ 4x-1 > 3x-4 \\ 3-2x < x-6 \end{cases}$

$$(e) \begin{cases} 3x + 2 < 7 - 2x \\ 48x < 3x + 10 \\ 11 - 2(x - 3) > 1 - 3(x - 5) \end{cases} \quad (f) \begin{cases} \frac{2x - 5}{1 - x} \leq -2 \\ \frac{x^2 + x + 3}{x + 1} > x \end{cases}$$

13. Resolva, em \mathbb{R} , as inequações:

$$\begin{array}{ll} (a) (3x + 3)(5x - 3) > 0 & (e) (6x - 1)(2x + 7) \geq 0 \\ (b) (4 - 2x)(5 + 2x) < 0 & (f) (5 - 2x)(-7x - 2) \leq 0 \\ (c) (5x + 2)(2 - x)(4x + 3) > 0 & (g) (3 - 2x)(4x + 1)(5x + 3) \geq 0 \\ (d) (3x + 2)(-3x + 4)(x - 6) < 0 & (h) (5 - 3x)(7 - 2x)(1 - 4x) \leq 0 \end{array}$$

14. Resolva, em \mathbb{R} , as inequações:

$$\begin{array}{lll} (a) (x + 3)^4 > 0 & (d) (1 - 7x)^5 > 0 & (g) (4 + 3x)^4 \geq 0 \\ (b) (3x + 8)^3 < 0 & (e) (3x + 5)^2 \geq 0 & \\ (c) (4 - 5x)^6 < 0 & (f) (5x + 1)^3 \leq 0 & (h) (3x - 8)^5 \leq 0 \end{array}$$

15. Resolva, em \mathbb{R} , as inequações:

$$\begin{array}{ll} (a) (5x + 4)^4(7x - 2)^3 \geq 0 & (c) (x + 6)^7(6x - 2)^4(4x + 5)^{10} \leq 0 \\ (b) (3x + 1)^3(2 - 5x)^5(x + 4)^8 > 0 & (d) (5x - 1)(2x + 6)^8(4 - 6x)^6 \geq 0 \end{array}$$

16. Resolva, em \mathbb{R} , a inequação $(3x - 2)^3(x - 5)^2(2 - x)x > 0$.

17. Resolva, em \mathbb{R} , as inequações:

$$(a) \frac{2x + 1}{x + 2} > 0 \quad (b) \frac{3x - 2}{3 - 2x} < 0 \quad (c) \frac{3 - 4x}{5x + 1} \geq 0$$

18. Resolva, em \mathbb{R} , as inequações:

$$\begin{array}{lll} (a) \frac{5x - 3}{3x - 4} > -1 & (c) \frac{6x}{x + 3} < 5 & (e) \frac{3x - 5}{2x - 4} \leq 1 \\ (b) \frac{x - 1}{x + 1} \geq 3 & (d) \frac{5x - 2}{3x + 4} < 2 & (f) \frac{x + 1}{x - 2} \geq 4 \end{array}$$

19. Resolva, em \mathbb{R} , as inequações:

$$\begin{array}{ll} (a) \frac{(1 - 2x)(3 + 4x)}{4 - x} > 0 & (c) \frac{(5x + 4)(4x + 1)}{5 - 4x} \geq 0 \\ (b) \frac{3x + 1}{(2x + 5)(5x + 3)} < 0 & (d) \frac{1 - 2x}{(5 - x)(3 - x)} \geq 0 \end{array}$$

Respostas

1. (a) $y > 0$, se $x \in \left(-\frac{3}{2}, +\infty\right)$;
 $y < 0$, se $x \in \left(-\infty, -\frac{3}{2}\right)$.

(b) $y > 0$, se $x \in \left(-\infty, \frac{2}{3}\right)$;
 $y < 0$, se $x \in \left(\frac{2}{3}, +\infty\right)$.

(c) $y > 0$, se $x \in (-\infty, 4)$;
 $y < 0$, se $x \in (4, +\infty)$.

(d) $y > 0$, se $x \in (-5, +\infty)$;
 $y < 0$, se $x \in (-\infty, -5)$.

(e) $y > 0$, se $x \in (-\infty, 6)$;
 $y > 0$, se $x \in (6, +\infty)$.

(f) $y > 0$, se $x \in \left(-\frac{9}{2}, +\infty\right)$;
 $y < 0$, se $x \in \left(-\infty, -\frac{9}{2}\right)$.

(g) $y > 0$, se $x \in \left(\frac{2}{3}, +\infty\right)$;
 $y < 0$, se $x \in \left(-\infty, \frac{2}{3}\right)$.

(h) $y > 0$, se $x \in (-\infty, 0)$;
 $y > 0$, se $x \in (0, +\infty)$.

2.

3. $x < 3$

4. $x > \frac{4}{3}$

5. (a) $x \geq -\frac{1}{5}$

(b) $x > \frac{1}{2}$

(c) $\forall x \in \mathbb{R}$

6. (a) $S = \{x \in \mathbb{R} | x > -4\}$

(b) $S = \{x \in \mathbb{R} | x \leq -10\}$

(c) $S = \left\{x \in \mathbb{R} | x > -\frac{3}{4}\right\}$

7.

8. (a) $S = \{x \in \mathbb{R} | x \geq 3\}$

(b) $S = \{x \in \mathbb{R} | x > -3\}$

(c) $S = \{x \in \mathbb{R} | x \geq 7\}$

(d) $S = \{x \in \mathbb{R} | x < 0\}$

(e) $S = \emptyset$

(f) $S = \mathbb{R}$

9.

10. (a) $S = \{x \in \mathbb{R} | x > 1\}$

(b) $S = \left\{x \in \mathbb{R} | x < \frac{1}{2}\right\}$

(c) $S = \left\{x \in \mathbb{R} | x > -\frac{2}{3}\right\}$

11. (a) $S = \left\{x \in \mathbb{R} | -\frac{1}{3} < x < \frac{5}{3}\right\}$

(b) $S = \left\{x \in \mathbb{R} | \frac{1}{2} \leq x < 4\right\}$

(c) $S = \left\{x \in \mathbb{R} | -\frac{1}{3} < x < 1\right\}$

(d) $S = \emptyset$

(e) $S = \left\{x \in \mathbb{R} | x < \frac{1}{3}\right\}$

(f) $S = \{x \in \mathbb{R} | x > 1\}$

12. (a) $S = \{x \in \mathbb{R} | 1 \leq x \leq 2\}$

(b) $S = \{x \in \mathbb{R} | x < -3\}$

(c) $S = \{x \in \mathbb{R} | 3 \leq x \leq 6\}$

(d) $S = \emptyset$

(e) $S = \left\{x \in \mathbb{R} | -1 < x < \frac{2}{9}\right\}$

(f) $S = \{x \in \mathbb{R} | -1 < x < 1\}$

13. (a) $S = \left\{x \in \mathbb{R} | x < -1 \text{ ou } x > \frac{3}{5}\right\}$

(b) $S = \left\{x \in \mathbb{R} | x < -\frac{5}{2} \text{ ou } x > 2\right\}$

(c) $S = \left\{x \in \mathbb{R} | x < -\frac{3}{4} \text{ ou } -\frac{2}{5} < x < 2\right\}$

(d) $S = \left\{x \in \mathbb{R} | -\frac{2}{3} < x < \frac{4}{3} \text{ ou } x > 6\right\}$

(e) $S = \left\{x \in \mathbb{R} | x \leq -\frac{7}{2} \text{ ou } x \geq \frac{1}{6}\right\}$

(f) $S = \left\{x \in \mathbb{R} | -\frac{2}{7} \leq x \leq \frac{5}{2}\right\}$

(g) $S = \left\{x \in \mathbb{R} | x \leq -\frac{3}{5} \text{ ou } -\frac{1}{4} \leq x \leq \frac{3}{2}\right\}$

- (h) $S = \left\{ x \in \mathbb{R} \mid \frac{1}{4} \leq x < \frac{5}{3} \text{ ou } x \geq \frac{7}{2} \right\}$
14. (a) $S = \{x \in \mathbb{R} \mid x \neq 3\}$
 (b) $S = \left\{ x \in \mathbb{R} \mid x < -\frac{8}{3} \right\}$
 (c) $S = \emptyset$
 (d) $S = \left\{ x \in \mathbb{R} \mid x < \frac{1}{7} \right\}$
 (e) $S = \mathbb{R}$
 (f) $S = \left\{ x \in \mathbb{R} \mid x \leq -\frac{1}{5} \right\}$
 (g) $S = \left\{ -\frac{4}{3} \right\}$
 (h) $S = \left\{ x \in \mathbb{R} \mid x \geq \frac{8}{3} \right\}$
15. (a) $S = \left\{ x \in \mathbb{R} \mid x \geq \frac{2}{7} \right\}$
 (b) $S = \left\{ x \in \mathbb{R} \mid -\frac{1}{3} < x < \frac{2}{5} \right\}$
 (c) $S = \{x \in \mathbb{R} \mid x \leq -6\} \cup \left\{ -\frac{5}{4}, \frac{1}{3} \right\}$
 (d) $S = \left\{ x \in \mathbb{R} \mid x \geq \frac{1}{5} \text{ ou } x = -3 \right\}$
16. $S = \left\{ x \in \mathbb{R} \mid x < 0 \text{ ou } \frac{2}{3} < x < 2 \right\}$
17. (a) $S = \left\{ x \in \mathbb{R} \mid x < -2 \text{ ou } x > -\frac{1}{2} \right\}$
 (b) $S = \left\{ x \in \mathbb{R} \mid x < \frac{2}{3} \text{ ou } x > \frac{3}{2} \right\}$
 (c) $S = \left\{ x \in \mathbb{R} \mid -\frac{1}{5} < x \leq \frac{3}{4} \right\}$
18. (a) $S = \left\{ x \in \mathbb{R} \mid x < \frac{7}{8} \text{ ou } x > \frac{4}{3} \right\}$
 (b) $S = \{x \in \mathbb{R} \mid -2 \leq x < -1\}$
 (c) $S = \{x \in \mathbb{R} \mid -3 < x < 15\}$
 (d) $S = \left\{ x \in \mathbb{R} \mid x < -10 \text{ ou } x > -\frac{4}{3} \right\}$
 (e) $S = \{x \in \mathbb{R} \mid 1 \leq x < 2\}$
 (f) $S = \{x \in \mathbb{R} \mid 2 < x \leq 3\}$
19. (a) $S = \left\{ x \in \mathbb{R} \mid -\frac{3}{4} < x < \frac{1}{2} \text{ ou } x > 4 \right\}$
 (b) $S = \left\{ x \in \mathbb{R} \mid x < -\frac{5}{2} \text{ ou } -\frac{3}{5} < x < -\frac{1}{3} \right\}$
 (c) $S = \left\{ x \in \mathbb{R} \mid x \leq -\frac{4}{5} \text{ ou } -\frac{1}{4} \leq x < \frac{5}{4} \right\}$
 (d) $S = \left\{ x \in \mathbb{R} \mid \frac{1}{2} \leq x < 3 \text{ ou } x > 5 \right\}$