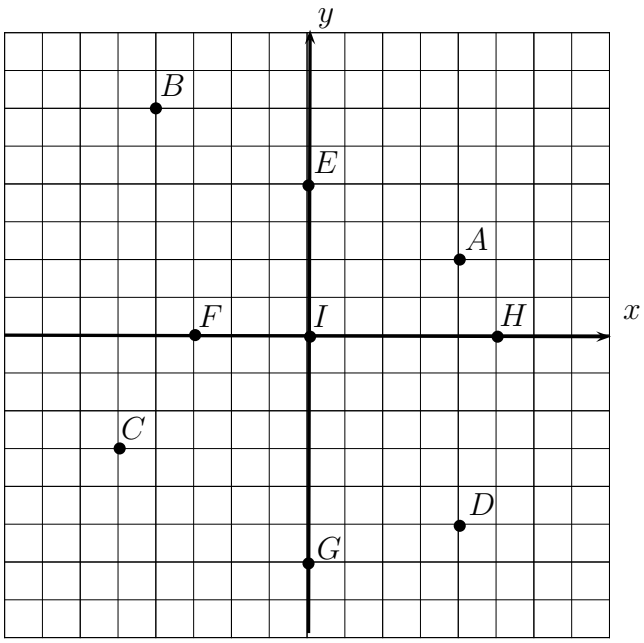
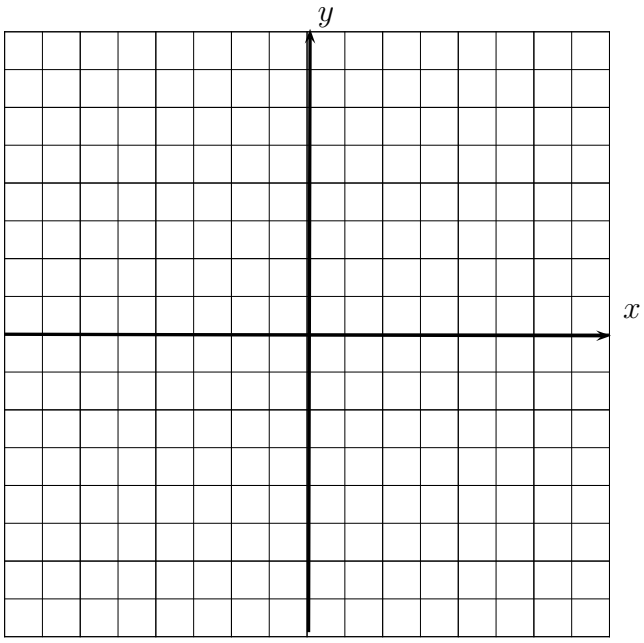
4ª LISTA DE EXERCÍCIOS

Prof. Ânderson Vieira

1. Dê as coordenadas de cada ponto do plano cartesiano abaixo.



2. Assinale no plano cartesiano os pontos: $A(2,-3)$, $B(0,-4)$, $C(-4,-5)$, $D(-1,0)$, $E(0,5)$, $F(5,4)$, $G(3,0)$, $H(-3,2)$, $I\left(\frac{1}{2},\frac{5}{2}\right)$



3. Dados os conjuntos

$A = \{1,3,4\}$ $B = \{-2,1\}$ $C = \{-1,0,2\}$

represente pelos elementos e pelo gráfico cartesiano os seguintes produtos:

(a) $A \times B$ (c) $A \times C$ (e) B^2

(b) $B \times A$ (d) $C \times A$ (f) C^2

4. Dados os conjuntos

   $A = \{x \in \mathbb{R} | 1 \leq x \leq 3\}$

   $B = \{x \in \mathbb{R} | -2 \leq x \leq 2\}$

   $C = \{x \in \mathbb{R} | -4 < x \leq 1\}$

   represente graficamente os seguintes produtos:

   (a) $A \times B$ (c) $B \times C$ (e) A^2

   (b) $A \times C$ (d) $C \times B$ (f) C^2

5. Dados os conjuntos $A = \{1, 2, 3, 4\}$ e $B = \{x \in \mathbb{R} | 1 \leq x \leq 4\}$, represente graficamente os conjuntos:

   (a) $A \times B$

   (b) $B \times A$

   (c) $(A \times B) \cup (B \times A)$

6. Sejam $F = \{1.2, 3, 4\}$ e $G = \{3, 4, 7\}$. Determine o número de elementos de $F \times G$.

7. Dados os conjunto $A = \left\{1, \frac{3}{2}\right\} \cup \{x \in \mathbb{R} | 2 < x < 3\}$ e $B = \{x \in \mathbb{R} | 1 \leq x \leq 2\}$, represente graficamente $A \times B$.

8. Seja $\mathbb{Z}$ o conjuntos dos números inteiros. Sejam ainda os conjunto $A = \{x \in \mathbb{Z} | -1 < x \leq 2\}$ e $B = \{3, 4, 5\}$. Qual é o número de elementos do conjunto $D = \{(x, y) \in A \times B | y \geq x + 4\}$?

9. I) Enumere os pares ordenados.

   II) Represente por meio de flechas.

   III) Faça o gráfico cartesiano das relações binárias de $A = \{-2, -1, 0, 1, 2\}$ em $B = \{-3, -2, -1, 1, 2, 3, 4\}$ definidas por:

   (a) $x\,R\,y \Leftrightarrow x + y = 2$

   (b) $x\,S\,y \Leftrightarrow x^2 = y$

   (c) $x\,T\,y \Leftrightarrow |x| = |y|$

   (d) $x\,V\,y \Leftrightarrow x + y > 2$

   (e) $x\,W\,y \Leftrightarrow (x - y)^2 = 1$

10. Dado o conjunto $A = \{1, 2, 3, 4, 5, 6\}$, enumere os pares ordenados e construa o gráfico cartesiano da relação R em R dada por

$$R = \{(x, y) \in A^2 | mdc(x, y) = 2\}$$

11. Seja o conjunto $A = \{1, 2, 3, 4, 5, 6\}$. Construa o gráfico cartesiano da relação R em A definida por

$$x\,R\,y \Leftrightarrow mdc(x, y) = 1^*$$

*x e y são primos entre si

12. Dado o conjunto $A = \{m \in \mathbb{Z} | -7 \leq m \leq 7\}$, construa o gráfico cartesiano da relação binária R em A definida por
$$x\,R\,y \Leftrightarrow x^2 + y^2 = 25$$

13. Estabeleça o domínio e a imagem das seguintes relações:
    (a) $\{(1,1),(1,3),(2,4)\}$
    (b) $\{(-2,4),(-1,1),(3,-7),(2,1)\}$
    (c) $\{(2,1),(1,-3),(5,\sqrt{2})\}$
    (d) $\{(1+\sqrt{2},\sqrt{2}),(1-\sqrt{3},1)\}$
    (e) $\left\{\left(3,\frac{1}{2}\right),\left(\frac{5}{2},-1\right),\left(\frac{3}{2},0\right)\right\}$

14. Estabeleça o domínio e a imagem das relações binárias do exercícios 9.

15. Sejam os conjuntos $A = \{-2,-1,0,1,2,3,4,5\}$, $B = \{-2,-1,0,1,2\}$ e R a relação binária de A em B definida por:
$$x\,R\,y \Leftrightarrow x = y^2$$
    (a) Enumere os pares ordenados de R.
    (b) Enumere os elementos do domínio e da imagem de R.
    (c) Faça o gráfico cartesiano de R.

16. Qual é o domínio da relação
$$f = \left\{(x,y) \in \mathbb{R} \times \mathbb{R} | y = \frac{2}{4-x^2}\right\}?$$

17. Se R é a relação binária de $A = \{x \in \mathbb{R} | 1 \leq x \leq 6\}$ em $B = \{y \in \mathbb{R} | 1 \leq y \leq 4\}$, definida por
$$x\,R\,y \Leftrightarrow x = 2y$$
    forneça
    (a) a representação cartesiana de $A \times B$;
    (b) a representação cartesiana de R;
    (c) o domínio e a imagem de R.

18. Se R e S são as relações binárias de $A = \{x \in \mathbb{Z} | -2 \leq x \leq 5\}$ em $B = \{y \in \mathbb{Z} | -2 \leq y \leq 3\}$ definidas por
$$x\,R\,y \Leftrightarrow 2 \text{ divide } (x-y)$$
$$x\,S\,y \Leftrightarrow (x-1)^2 = (y-2)^2$$
    forneça
    (a) as representações cartesianas de R e de S;
    (b) o domínio e a imagem de R e de S;
    (c) $R \cap S$.

19. Enumere os elementos de R^{-1}, relação inversa de R, nos seguintes casos

(a) $R = \{(1,2),(3,1),(2,3)\}$

(b) $R = \{(1,-1),(2,-1),(3,-1),(-2,1)\}$

(c) $R = \{(-3,-2),(1,3),(-2-3),(3,1)\}$

20. Enumere os elementos e esboce os gráficos de R e R^{-1}, relações binárias em $A = \{x \in \mathbb{N} | x \leq 10\}$, nos seguintes casos

(a) $R = \{(x,y) \in A^2 | x + y = 8\}$

(b) $R = \{(x,y) \in A^2 | x + 2y = 10\}$

(c) $R = \{(x,y) \in A^2 | y = (x-3)^2 + 1\}$

(d) $R = \{(x,y) \in A^2 | y = 2^x\}$

21. Dados os conjuntos $A = \{x \in \mathbb{R} | 1 \leq x \leq 6\}$, $B = \{y \in \mathbb{R} | 2 \leq y \leq 10\}$ e as seguintes relações binárias

(a) $R = \{(x,y) \in A \times B | x = y\}$

(b) $S = \{(x,y) \in A \times B | y = 2x\}$

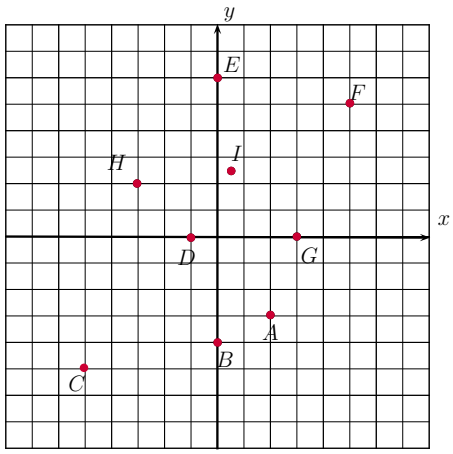
(c) $T = \{(x,y) \in A \times B | y = x + 2\}$

(d) $V = \{(x,y) \in A \times B | x + y = 7\}$

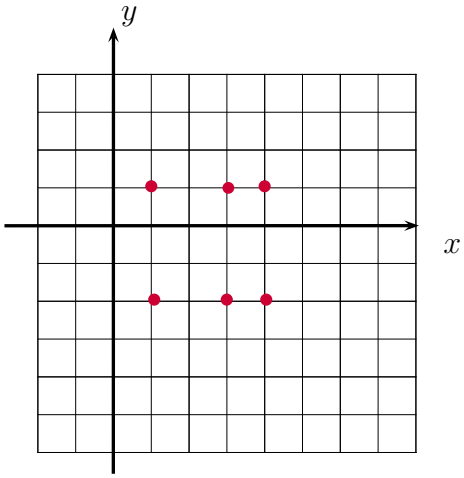
dê o gráfico cartesiano dessas relações e das respectivas relações inversas.

Respostas

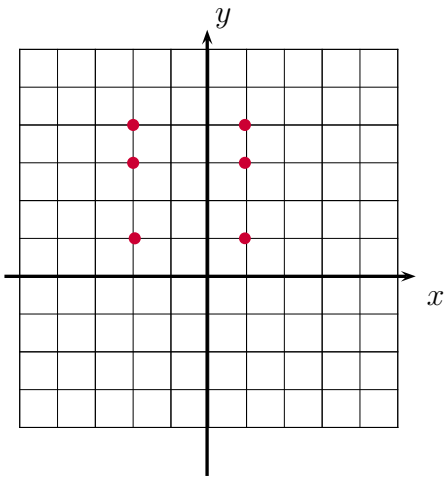
1. $A(4,2)$, $B(-4,6)$, $C(-5,-3)$, $D(4,-5)$, $E(0,4)$, $F(-3,-0)$, $G(0,-6)$, $H(5,0)$, $I(0,0)$.

2. 

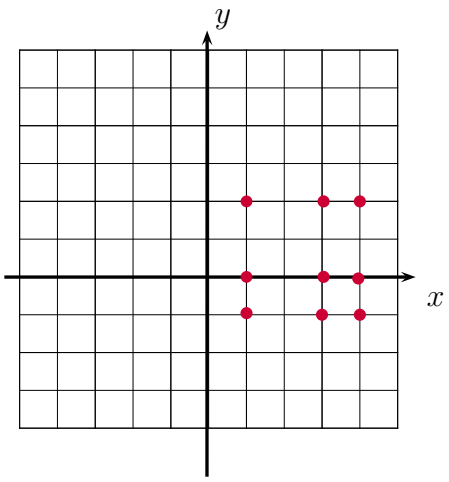
3. (a) $A \times B = \{(1,-2),(1,1),(3,-2),(3,1),(4,-2),(4,1)\}$



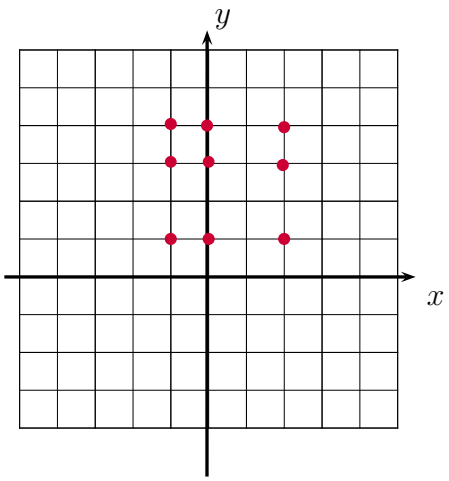
(b) $B \times A = \{(-2,1),(-2,3),(-2,4),(1,1),(1,3),(1,4)\}$



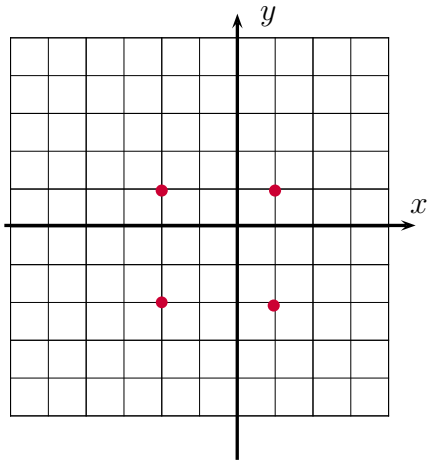
(c) $A \times C = \{(1,-1),(1,0),(1,2),(3,-1),(3,0),(3,2),(4,-1),(4,0),(4,2)\}$



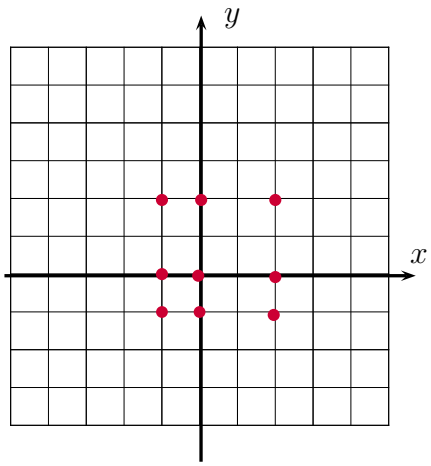
(d) $C \times A = \{(-1,1),(-1,3),(-1,4),(0,1),(0,3),(0,4),(2,1),(2,3),(2,4)\}$



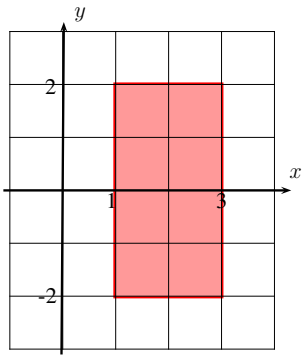
(e) $B^2 = \{(-2,-2),(-2,1),(1,-2),(1,1)\}$



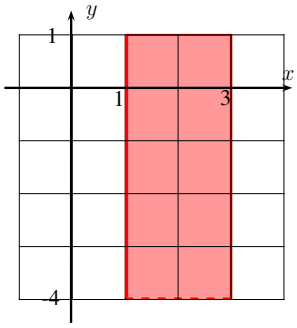
(f) $C^2 = \{(-1,-1),(-1,0),(-1,2),(0,-1),(0,0),(0,2),(2,-1),(2,0),(2,2)\}$



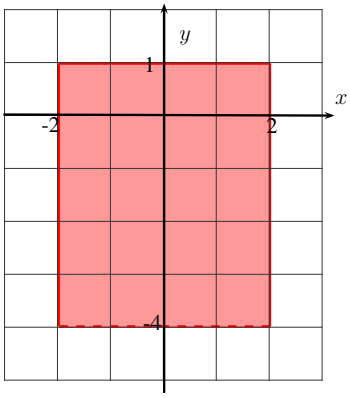
4. (a) $A \times B$



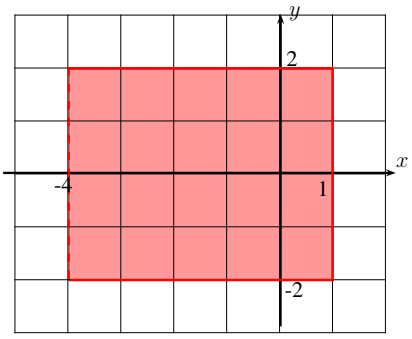
(b) $A \times C$



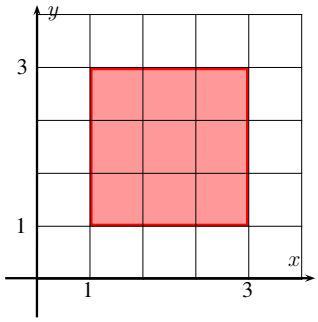
(c) $B \times C$



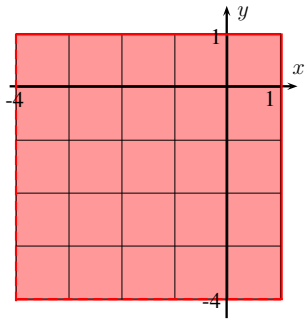
(d) $C \times B$



(e) A^2

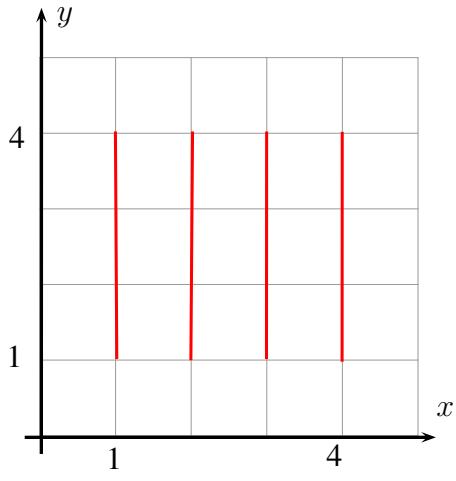


(f) C^2

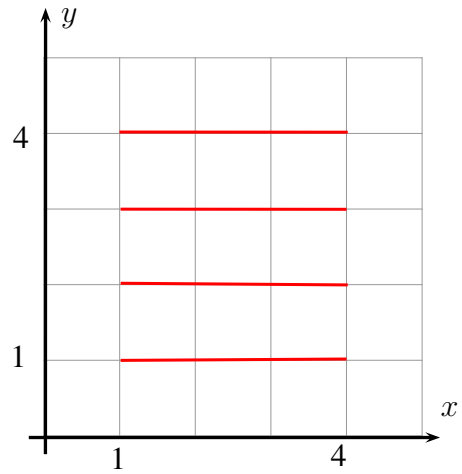


5. Dados os conjuntos $A = \{1, 2, 3, 4\}$ e $B = \{x \in \mathbb{R} | 1 \leq x \leq 4\}$, represente graficamente os conjuntos:

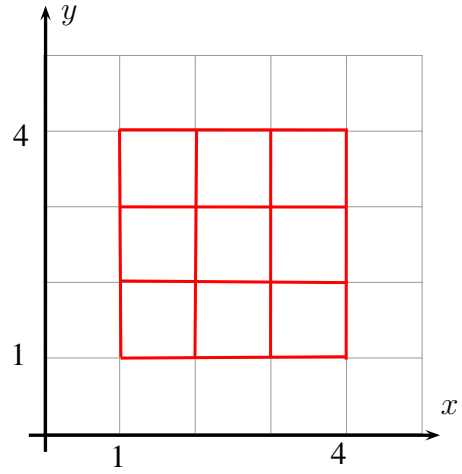
(a) $A \times B$



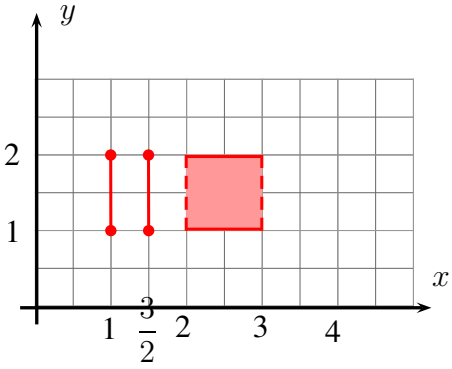
(b) $B \times A$



(c) $(A \times B) \cup (B \times A)$



6. $n(F \times G) = 12$

7. 

8. $n(D) = 3$

9. (a) $R = \{(-2, 4), (-1, 3), (0, 2), (1, 1)\}$

   (b) $S = \{(-2, 4), (2, 4), (-1, 1), (1, 1)\}$

   (c) $T = \{(-2, -2), (-2, 2), (-1, -1), (-1, 1), (1, -1), (1, 1), (2, -2), (2, 2)\}$

   (d) $V = \{(-1, 4), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

   (e) $W = \{(-2, -3), (-2, -1), (-1, -2), (0, -1), (0, 1), (1, 2), (2, 1), (2, 3)\}$

10. $R = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 6), (6, 2), (6, 4)\}$

11. $R = \{(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (3, 5), (4, 1), (4, 3), (4, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 5)\}$

12. $R = \{(-5, 0), (-4, -3), (-4, 3), (-3, -4), (-3, 4), (0, -5), (0, 5), (3, -4), (3, 4), (4, -3), (4, 3), (-5, 0)\}$

13. (a) $D = \{1, 2\}$ e $Im = \{1, 3, 4\}$

    (b) $D = \{-2, -1, 2, 3\}$ e $Im = \{-7, 1, 4\}$

    (c) $D = \{1, 2, 5\}$ e $Im = \{-3, 1\sqrt{2}\}$

    (d) $D = \{1 - \sqrt{3}, 1 + \sqrt{2}\}$ e $Im = \{1, \sqrt{2}\}$

    (e) $D = \left\{3, \frac{5}{2}, \frac{3}{2}\right\}$ e $Im = \left\{\frac{1}{2}, -1, 0\right\}$

14. (a) $D(R) = \{-2, -1, 0, 1\}$ e $Im(R) = \{1, 2, 3, 4\}$

    (b) $D(S) = \{-2, -1, 1, 2\}$ e $Im(S) = \{1, 4\}$

    (c) $D(T) = \{-2, -1, 1, 2\}$ e $Im(T) = \{-2, -1, 1, 2\}$

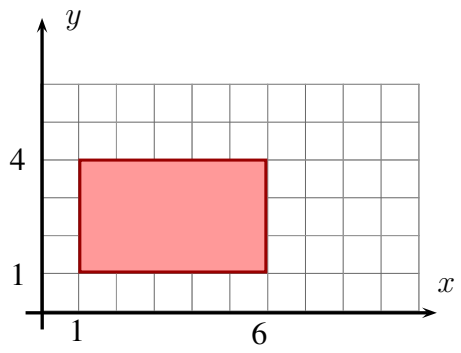
    (d) $D(V) = \{-1, 0, 1, 2\}$ e $Im(V) = \{1, 2, 3, 4\}$

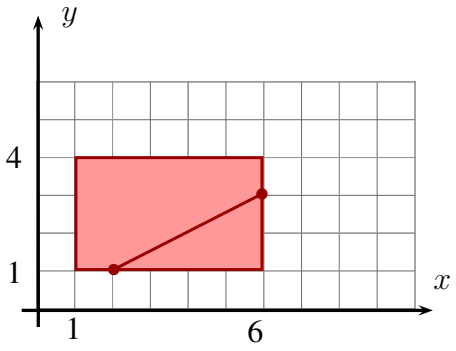
    (e) $D(W) = \{-2, -1, 0, 1, 2\}$ e $Im(W) = \{-3, -2, -1, 1, 2, 3\}$

15. (a) $R = \{(0, 0), (1, -1), (1, 1), (4, -2), (4, 2)\}$

    (b) $D(R) = \{0, 1, 4\}$ e $Im(R) = \{-2, -1, 0, 1, 2\}$

16. $D_f = \{x \in \mathbb{R} | x \neq -2 \, e \, x \neq 2\}$

17. (a) 



(b)

(c) $D(R) = \{x \in \mathbb{R} | 2 \leq y \leq 6\}$ e $Im(R) = \{y \in \mathbb{R} | 1 \leq y \leq 3\}$

18. (a) $R = \{(-2,-2),(-2,0),(-2,2),$
$(-1,-1),(-1,1),(-1,3),(0,-2),(0,0),$
$(0,2),(1,-1),(1,1),(1,3),(2,-2),(2,0),$
$(2,2),(3,-1),(3,1),(3,3),(4,-2),(4,0),$
$(4,2),(5,-1),(5,1),(5,3)\}$
$S = \{(-2,-1),(-1,0),(0,1),(0,3),(1,2),$
$(1,1),(1,3),(3,1),(4,0),(4,-1),(5,-2)\}$;

(b) $D(R) = \{-2,-1,0,1,2,3,4,5\} = D(S)$
$Im(R) = \{-2,-1,0,1,2,3\} = Im(S)$

(c) $R \cap S = \emptyset$.

19. (a) $R^{-1} = \{(2,1),(1,3),(3,2)\}$

(b) $R^{-1} = \{(-1,1),(-1,2),(-1,3),(1,-2,)\}$

(c) $R^{-1} = \{(-2,-3),(3,1),(-3,2),(1,3)\}$

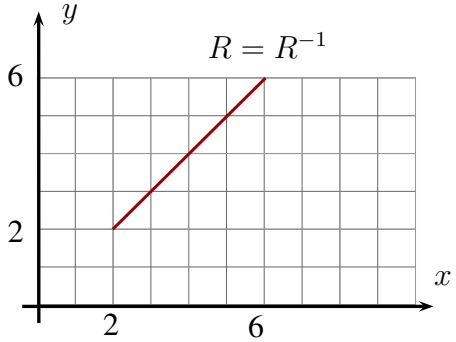
20. (a) $R = R^{-1} = \{(0,8),(1,7),(2,6),(3,5),$
$(4,4),(5,3),(6,2),(7,1),(8,0)\}$

(b) $R = \{(0,5),(2,4),(4,3),(6,2),(8,1),$
$(10,0)\}$ $R^{-1} = \{(5,0),(4,2),(3,4),(2,6),(1,8),$
$(0,10)\}$

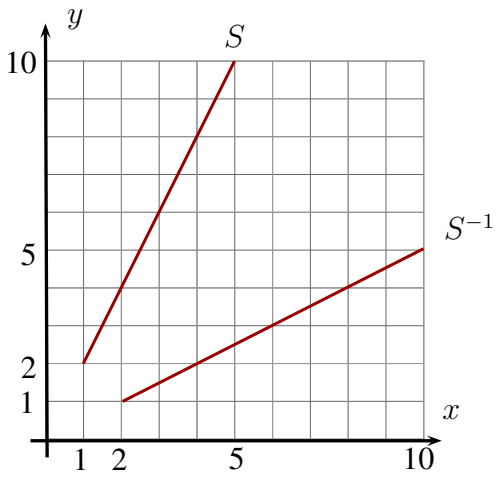
(c) $R = \{(0,10),(1,5),(2,2),(3,1),(4,2),$
$(5,5),(6,10)\}$
$R^{-1} = (10,0),(5,1),(2,2),(1,3),(2,4),(5,5),(10,6)\}$

(d) $R = \{(0,1),(1,2),(2,4),(3,8)\}$
$R^{-1} = \{(1,0),(2,1),(4,2),(8,3)\}$

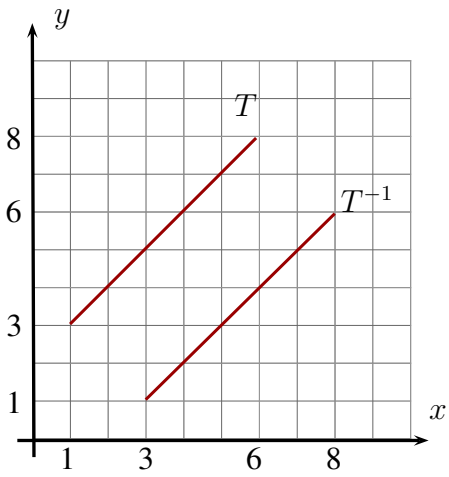
21. (a) $R = \{(x,y) \in A \times B | x = y\}$



(b) $S = \{(x,y) \in A \times B | y = 2x\}$



(c) $T = \{(x,y) \in A \times B | y = x + 2\}$



(d) $V = \{(x,y) \in A \times B | x + y = 7\}$